

Relativistic Covariance of Light-Front Few-Body Systems in Hadron Physics

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Abstract We study the light-front covariance of a vector-meson decay constant using a manifestly covariant fermion field theory model in $(3 + 1)$ dimensions. The light-front zero-mode issues are analyzed in terms of polarization vectors and method of identifying the zero-mode operator and of obtaining the light-front covariant decay constant is discussed.

Keywords Weak Decay · Decay Constant · Light-Front Zero-Mode

1 Introduction

Mesonic weak transition form factors and decay constants are two of the most important ingredients in studying weak decays of mesons, which enter in various decay rates. Many theoretical efforts were undertaken to calculate these observables. The light-front quark model (LFQM) based on the LF dynamics (LFD) has been quite successful in describing various exclusive decays of mesons [1, 2, 3, 4, 5, 6, 7]. However, one should also realize that the success of LFD in hadron physics cannot be realized unless the treacherous points in LFD such as the zero-mode contributions in the hadron form factors [1, 2, 3, 4, 5, 6, 7] are well taken care of with proper methods.

In this paper, we study the LF covariance of the vector-meson decay constant using a manifestly covariant fermion field theory model in $(3 + 1)$ dimensions. Although the LF covariant issue for the vector meson decay constant has been raised by Jaus [1] some time ago, systematic analyses of the zero-modes depending on

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different polarization vectors have not yet been explored much. Here, we attempt to systematically investigate the LF zero-mode issues in terms of polarization vectors of a vector meson and show a method of identifying the zero-mode operator to obtain the LF covariant decay constant even in the case that there exists a zero-mode contribution.

2 Manifestly Covariant Calculation

The decay constant f_V of a vector meson of mass M and bound state of a quark q of mass m_1 , and an antiquark \bar{q} of mass m_2 , is defined by the matrix element of the vector current

$$\langle 0 | \bar{q} \gamma^\mu q | V(P, h) \rangle = f_V M \epsilon^\mu(h), \quad (1)$$

where the polarization vector ϵ of a vector meson satisfies the Lorentz condition $\epsilon \cdot P = 0$.

The matrix element $A_h^\mu \equiv \langle 0 | \bar{q} \gamma^\mu q | V(P, h) \rangle$ is given in the one-loop approximation as a momentum integral

$$A_h^\mu = N_c \int \frac{d^4 k}{(2\pi)^4} \frac{H_V}{N_p N_k} \text{Tr} [\gamma^\mu (\not{p} + m_1) \Gamma \cdot \epsilon(h) (-\not{k} + m_2)], \quad (2)$$

where $N_p = p^2 - m_1^2 + i\varepsilon$ and $N_k = k^2 - m_2^2 + i\varepsilon$ with $p = P - k$ and N_c denotes the number of colors. For simplicity in regularizing the covariant loop, we take $n = 2$ in a multipole ansatz $H_V = g/(N_\Lambda)^n$ for the $q\bar{q}$ bound-state vertex function of a vector meson, where $N_\Lambda = p^2 - \Lambda^2 + i\varepsilon$, and g and Λ are constant parameters. The vector meson vertex operator Γ^μ in the trace term $S_h^\mu = \text{Tr} [\gamma^\mu (\not{p} + m_1) \Gamma \cdot \epsilon(h) (-\not{k} + m_2)]$ is given by $\Gamma^\mu = \gamma^\mu - (p - k)^\mu/D$. In this work, we shall analyze for the two cases of Γ^μ , i.e., (1) $\Gamma^\mu = \gamma^\mu$ (i.e. $1/D = 0$) case and (2) Γ^μ with constant D factor (i.e. $D = D_{\text{con}} = M + m_1 + m_2$) case for the explicit comparison between the manifestly covariant calculation and the LF one. The manifestly covariant result is given by

$$f_V^{\text{Cov}} = \frac{N_c g}{4\pi^2 M} \int_0^1 dx \int_0^{1-x} dy (1 - x - y) \left\{ \frac{y(1-y)M^2 + m_1 m_2}{C_{\text{cov}}^2} - \frac{1 + \frac{m_1 + m_2}{D_{\text{con}}}}{C_{\text{cov}}} \right\}, \quad (3)$$

where $C_{\text{cov}} = y(1-y)M^2 - x m_1^2 - y m_2^2 - (1-x-y)\Lambda^2$.

3 Light-Front Calculation

Performing the LF calculation in parallel with the manifestly covariant calculation, we use two different approaches, i.e. (1) plus component ($\mu = +$) of the currents with the longitudinal polarization $\epsilon(0)$ and (2) perpendicular components ($\mu = \perp$) of the currents with the transverse polarization $\epsilon(\pm)$, to obtain the decay constant. The explicit form of polarization vectors of a vector meson is given in [6].

By the integration over k^- in Eq. (2) and closing the contour in the lower half of the complex k^- plane, one picks up the residue at $k^- = k_{\text{on}}^-$ (on shell value

of k^- in the region $0 < k^+ < P^+$ (or $0 < x < 1$). Thus, the Cauchy integration formula for the k^- integral in Eq. (2) gives

$$A_h^\mu = \frac{N_c}{16\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2\mathbf{k}_\perp \chi(x, \mathbf{k}_\perp) S_h^\mu(k^- = k_{\text{on}}^-), \quad (4)$$

where $S_h^\mu(k^- = k_{\text{on}}^-)$ is the result of the trace when $k^- = k_{\text{on}}^-$ and $\chi(x, \mathbf{k}_\perp) = g/[x^3(M_0^2 - M_0^2)(M^2 - M_A^2)^2]$ with $M_{0(A)}^2 = [\mathbf{k}_\perp^2 + m_1^2(\Lambda^2)]/x + [\mathbf{k}_\perp^2 + m_2^2]/(1-x)$.

Firstly, using $\mu = +$ with the longitudinal polarization vector $\epsilon(0)$ in Eq. (4), the decay constant is obtained from the relation

$$f_V^{(h=0)} = \frac{A_{h=0}^+}{\epsilon^+(0)M}. \quad (5)$$

For the purpose of analyzing zero-mode contribution to the decay constant, we denote the decay constant as $[f_V^{(h=0)}]_{\text{val}}$ when the matrix element $A_{h=0}^+$ is obtained for $k^- = k_{\text{on}}^-$ in the region of $0 < x < 1$. Comparing $[f_V^{(h=0)}]_{\text{val}}$ with the manifestly covariant result f_V^{Cov} , we find that $[f_V^{(h=0)}]_{\text{val}}$ is exactly the same as f_V^{Cov} when $\Gamma^\mu = \gamma^\mu$ (or $1/D = 0$) is used. The same observation has also been made in [4]. However, $f_V^{(h=0)}$ is different from f_V^{Cov} when $D = D_{\text{con}}$ is used. The difference between the two results, i.e. $f_V^{\text{Cov}} - [f_V^{(h=0)}]_{\text{val}}$, corresponds to the zero-mode contribution $[f_V^{(h=0)}]_{\text{Z.M.}}$ to the full solution $[f_V^{(h=0)}]_{\text{Full}} = [f_V^{(h=0)}]_{\text{val}} + [f_V^{(h=0)}]_{\text{Z.M.}}$.

As in the case of zero-mode contribution to the weak transition form factors for semileptonic $P \rightarrow P$ and $P \rightarrow V$ decays [5,6], the zero-mode contribution to $f_V^{(h=0)}$ comes (if exists) from the singular p^- (or equivalently $1/x$) term in $S_{h=0}^+$ in the limit of $x \rightarrow 0$ when $p^- = p_{\text{on}}^-$. For the case of $D = D_{\text{con}}$, we find the following singular term in $S_{h=0}^+$ as follows

$$\lim_{x \rightarrow 0} S_{h=0}^+(p^- = p_{\text{on}}^-) = 4m_1 \frac{\epsilon^+(0)p^-}{D_{\text{con}}}. \quad (6)$$

As we presented in the weak transition form factor calculation [5,6], we identify the zero-mode operator $[S_{h=0}^+]_{\text{Z.M.}}$ by replacing p^- with the operator $-Z_2$ [1,5,6]: i.e. $[S_{h=0}^+]_{\text{Z.M.}} = 4m_1 \frac{\epsilon^+(0)(-Z_2)}{D_{\text{con}}}$, where $Z_2 = x(M^2 - M_0^2) + m_1^2 - m_2^2 + (1-2x)M^2$. The zero-mode contribution to the matrix element $A_{h=0}^+$ is given by

$$[A_{h=0}^+]_{\text{Z.M.}} = \frac{N_c}{16\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2\mathbf{k}_\perp \chi(x, \mathbf{k}_\perp) [S_{h=0}^+]_{\text{Z.M.}}, \quad (7)$$

and the corresponding zero-mode contribution to the decay constant is obtained as $[f_V^{(h=0)}]_{\text{Z.M.}} = [A_{h=0}^+]_{\text{Z.M.}}/(\epsilon^+(0)M)$. Finally, we obtain the full result of the decay constant for the longitudinal polarization as

$$[f_V^{(h=0)}]_{\text{Full}} = \frac{N_c}{4M\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2\mathbf{k}_\perp \chi(x, \mathbf{k}_\perp) \left\{ x(1-x)M^2 + \mathbf{k}_\perp^2 + m_1m_2 + (m_1 + m_2) \frac{x[\mathbf{k}_\perp^2 + m_2^2 - (1-x)^2M^2]}{(1-x)D_{\text{con}}} \right\}. \quad (8)$$

It can be checked that Eq. (8) is identical to the manifestly covariant result of Eq. (3).

Secondly, using $\mu = \perp$ with the transverse polarization vector $\epsilon(+)$, the decay constant is obtained from the relation

$$f_V^{(h=1)} = \frac{A_{h=1}^\perp \cdot \epsilon_\perp^*(+)}{M}. \quad (9)$$

In this case, the decay constant $f_V^{(h=1)}$ receives the zero mode from the simple vertex γ^\perp term but not from the term including the D factor. Following the same procedure as for the case of $f_V^{(h=0)}$, we find the following singular term in $S_{h=1}^\perp$ as

$$\lim_{x \rightarrow 0} S_{h=1}^\perp(p^- = p_{\text{on}}^-) = 2p^- \epsilon_\perp(+), \quad (10)$$

and thus the corresponding zero-mode operator is given by $[S_{h=1}^\perp]_{\text{Z.M.}} = 2(-Z_2)\epsilon_\perp(+)$. Finally, we obtain the full result of the decay constant as follows

$$\begin{aligned} [f_V^{(h=1)}]_{\text{Full}} &= \frac{N_c}{4M\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2\mathbf{k}_\perp \chi(x, \mathbf{k}_\perp) \\ &\times \left\{ xM_0^2 - m_1(m_1 - m_2) - \mathbf{k}_\perp^2 + \frac{(m_1 + m_2)}{D} \mathbf{k}_\perp^2 \right\}. \end{aligned} \quad (11)$$

From Eq. (11) one can check that that $[f_V^{(h=1)}]_{\text{Full}}$ is the same as $[f_V^{(h=0)}]_{\text{Full}}$ [Eq. (8)], which confirm the result obtained by Jaus [1].

4 Conclusion

In this work, we investigate the LF zero-mode issue for the vector meson decay constant using two different polarization vectors of a vector meson. We find that the decay constant obtained from transverse polarization vectors cannot avoid the zero-mode even at the level of model-independent simple vector meson vertex, i.e. $\Gamma^\mu = \gamma^\mu$. Although the decay constant obtained from longitudinal polarization vector may receive a zero-mode depending on a model-dependent form of D factor, it is immune to the zero-mode at the level of simple vertex. The independence of the decay constant on the polarization vectors is also explicitly shown. Our results do not depend on the value of $n(\geq 2)$ in the multipole ansatz and may give an important guidance on a more realistic model building.

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